[2] $2\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 3x^2 \sum_{n=1}^{\infty} n a_n x^{n-1} - 6x \sum_{n=0}^{\infty} a_n x^n = 0$

$$\sum_{n=2}^{\infty} 2n(n-1)a_n x^{n-2} - \sum_{n=1}^{\infty} 3n a_n x^{n+1} - \sum_{n=0}^{\infty} 6a_n x^{n+1} = 0$$

$$\sum_{n=-1}^{\infty} 2(n+3)(n+2)a_{n+3} x^{n+1} - \sum_{n=1}^{\infty} 3n a_n x^{n+1} - \sum_{n=0}^{\infty} 6 a_n x^{n+1} = 0$$

$$2(2)(1)a_2 + 2(3)(2)a_3 x - 6a_0 x$$

$$+ \sum_{n=1}^{\infty} \left[2(n+3)(n+2)a_{n+3} - (3n+6)a_n\right] x^{n+1} = 0$$

$$4a_2 + (12a_3 - 6a_0)x + \sum_{n=1}^{\infty} \left[2(n+3)(n+2)a_{n+3} - 3(n+2)a_n\right] x^{n+1} = 0$$

$4a_2 = 0 \rightarrow a_2 = 0$

$12a_3 - 6a_0 = 0 \rightarrow a_3 = \frac{1}{2}a_0$

$2(n+3)\cancel{(n+2)}a_{n+3} - 3\cancel{(n+2)}a_n = 0 \rightarrow a_{n+3} = \dfrac{3}{2(n+3)}a_n$ FOR $n \geq 1$

LET $a_0 = 1$, $a_1 = 0 = a_4 = a_7 = \cdots$

$a_2 = 0 = a_5 = a_8 = \cdots$

$a_3 = \frac{1}{2}a_0 = \frac{1}{2}$

$n=3$: $a_6 = \dfrac{3}{2(6)}a_3 = \dfrac{3}{2^2 \cdot 6}$

$n=6$: $a_9 = \dfrac{3}{2(9)}a_6 = \dfrac{3^2}{2^3 \cdot 6 \cdot 9}$

$n=9$: $a_{12} = \dfrac{3}{2(12)}a_9 = \dfrac{3^3}{2^4 \cdot 6 \cdot 9 \cdot 12}$

$$y_1 = 1 + \frac{1}{2}x^3 + \frac{3}{2^2 \cdot 6}x^6 + \frac{3^2}{2^3 \cdot 6 \cdot 9}x^9 + \frac{3^3}{2^4 \cdot 6 \cdot 9 \cdot 12}x^{12} + \cdots$$

$$= 1 + \frac{3}{2 \cdot 3}x^3 + \frac{3^2}{2^2 \cdot 3 \cdot 6}x^6 + \frac{3^3}{2^3 \cdot 3 \cdot 6 \cdot 9}x^9 + \frac{3^4}{2^4 \cdot 3 \cdot 6 \cdot 9 \cdot 12}x^{12} + \cdots$$

$n=0 \quad 1 \quad 2 \quad 3 \quad 4$

$$= 1 + \sum_{n=1}^{\infty} \frac{3^n}{2^n \cdot 3 \cdot 6 \cdot 9 \cdots 3n} x^{3n}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{3^n}{2^n 3^n (n!)} x^{3n}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1}{2^n \cdot n!} x^{3n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2^n \cdot n!} x^{3n}$$

LET $a_0 = 0, a_1 = 1,$

$a_2 = 0 = a_5 = a_8 = \cdots$

$a_3 = \frac{1}{2}a_0 = 0 = a_6 = a_9 = \cdots$

$n=1: a_4 = \frac{3}{2(4)}a_1 = \frac{3}{2 \cdot 4}$

$n=4: a_7 = \frac{3}{2(7)}a_4 = \frac{3^2}{2^2 \cdot 4 \cdot 7}$

$n=7: a_{10} = \frac{3}{2(10)}a_7 = \frac{3^3}{2^3 \cdot 4 \cdot 7 \cdot 10}$

$$y_2 = x + \frac{3}{2\cdot 4}x^4 + \frac{3^2}{2^2\cdot 4\cdot 7}x^7 + \frac{3^3}{2^3\cdot 4\cdot 7\cdot 10}x^{10} + \cdots$$

$n=0$ $\quad 1 \quad 2 \quad 3$

$$= \sum_{n=0}^{\infty} \frac{3^n}{2^n(1\cdot 4\cdot 7\cdots(3n+1))} x^{3n+1}$$

$$y = c_1 y_1 + c_2 y_2$$

[3] $y'' + \underbrace{\frac{x-3}{x^2-2x}}\, y' - \underbrace{\frac{1}{x^2-2x}}\, y = 0$

BOTH DISCONT @ $x=0$

$\lim_{x\to 0} x \cdot \frac{x-3}{x^2-2x} = \lim_{x\to 0} \frac{x-3}{x-2} = \frac{3}{2}$ AND $\lim_{x\to 0} x^2 \cdot \frac{-1}{x^2-2x} = \lim_{x\to 0} -\frac{x}{x-2} = 0$ BOTH EXIST

$x=0$ IS A REGULAR SINGULAR POINT $\longrightarrow$ USE METHOD OF FROBENIUS

$$(x^2-2x)\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n x^{n+r-2} + (x-3)\sum_{n=0}^{\infty}(n+r)a_n x^{n+r-1} - \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n x^{n+r} - \sum_{n=0}^{\infty} 2(n+r)(n+r-1)a_n x^{n+r-1}$$
$$+ \sum_{n=0}^{\infty}(n+r)a_n x^{n+r} \qquad - \sum_{n=0}^{\infty} 3(n+r)a_n x^{n+r-1} \qquad = 0$$
$$- \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$\sum_{n=0}^{\infty}\left[(n+r)(n+r-1)+(n+r)-1\right]a_n x^{n+r} - \sum_{n=-1}^{\infty}\left[2(n+r+1)(n+r)+3(n+r+1)\right]a_{n+1}x^{n+r} = 0$$

$$-\left[2r(r-1)+3r\right]a_0 x^{r+1} + \sum_{n=0}^{\infty}\left[(n+r+1)(n+r-1)a_n - (2n+2r+3)(n+r+1)a_{n+1}\right]x^{n+1} = 0$$

$-(2r(r-1)+3r)=0 \longrightarrow r(2r-2+3)=0 \longrightarrow r(2r+1)=0 \longrightarrow r=0, -\frac{1}{2}$

$(\cancel{n+r+1})(n+r-1)a_n-(2n+2r+3)(\cancel{n+r+1})a_{n+1}=0$ FOR ALL $n\geq 0$

$a_{n+1}=\frac{n+r-1}{2n+2r+3}a_n$ FOR ALL $n\geq 0$

$r=0$: $a_{n+1}=\frac{n-1}{2n+3}a_n$

LET $a_0=1$

$n=0: a_1=\frac{-1}{3}a_0=-\frac{1}{3}$

$n=1: a_2=0=a_3=a_4=\cdots$

$y_1=1-\frac{1}{3}x$

$r=-\frac{1}{2}$: $a_{n+1}=\frac{n-\frac{3}{2}}{2n+2}a_n=\frac{2n-3}{4n+4}a_n$

LET $a_0=1$

$n=0: a_1=-\frac{3}{4}a_0=-\frac{3}{4}$

$n=1: a_2=-\frac{1}{8}a_1=\frac{1\cdot 3}{4\cdot 8}$

$n=2: a_3=\frac{1}{12}a_2=\frac{1\cdot 3\cdot 1}{4\cdot 8\cdot 12}$

$n=3: a_4=\frac{3}{16}a_3=\frac{1\cdot 3\cdot 1\cdot 3}{4\cdot 8\cdot 12\cdot 16}$

$n=4: a_5=\frac{5}{20}a_4=\frac{1\cdot 3\cdot 1\cdot 3\cdot 5}{4\cdot 8\cdot 12\cdot 16\cdot 20}$

$$y_2=\underset{n=0}{1}-\underset{1}{\frac{3}{4}x}+\underset{2}{\frac{1\cdot 3}{4\cdot 8}x^2}+\underset{3}{\frac{1\cdot 3\cdot 1}{4\cdot 8\cdot 12}x^3}+\underset{4}{\frac{1\cdot 3\cdot 1\cdot 3}{4\cdot 8\cdot 12\cdot 16}x^4}+\underset{5}{\frac{1\cdot 3\cdot 1\cdot 3\cdot 5}{4\cdot 8\cdot 12\cdot 16\cdot 20}x^5}+\cdots$$

$$\longrightarrow y_2=1+\sum_{n=1}^{\infty}\frac{(-3)(-1)\cdot 1\cdot 3\cdot 5\cdots(2n-5)}{4\cdot 8\cdot 12\cdots(4n)}x^n$$

$$=1+\sum_{n=1}^{\infty}\frac{3\cdot\frac{(2(n-2))!}{2^{n-2}(n-2)!}}{4^n\cdot n!}x^n$$

$$=1+\sum_{n=1}^{\infty}\frac{3((2n-4)!)}{2^{n-2}(n-2)!\,2^{2n}n!}x^n$$

$$=1+\sum_{n=1}^{\infty}\frac{3((2n-4)!)}{2^{3n-2}(n-2)!\,n!}x^n$$